

# How Quickly Things Go to 0 as $n$ goes to $\infty$

Math 112

Use this to help sharpen your intuition for the last problem on the Basic Skills Exam.

If  $(a_n)_{n=1}^{\infty}$  and  $(b_n)_{n=1}^{\infty}$  are sequences with  $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n = 0$ , we can say that  $(a_n)_{n=1}^{\infty}$  goes to zero faster than  $(b_n)_{n=1}^{\infty}$  if  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$ .

|        |              |  |
|--------|--------------|--|
|        | $a_n$        |  |
| Faster |              |  |
|        | $1/n^n$      | } Fast enough for $\sum a_n$ to converge |
|        | $1/n!$       |  |
|        | $2^n/n!$     |  |
|        | $3^n/n!$     |  |
|        | $1/3^n$      |  |
|        | $1/2^n$      |  |
|        | $n/2^n$      |  |
|        | $n^2/2^n$    |  |
|        | $1/n^3$      |  |
|        | $1/n^2$      |  |
|        | $1/n^{3/2}$  |  |
|        | $1/n$        |  |
|        | $1/\sqrt{n}$ |  |
|        | $1/\ln n$    |  |
| Slower |              |  |

Question: Where does  $\frac{n!}{n^n}$  fit into this?